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NOTE ON THE INTERPRETATION OF WAKE-SURVEY DATA AND ITS
USE IN THE ESTIMATION OF INDUCED DRAG
DUE TO IRREGULARITIES

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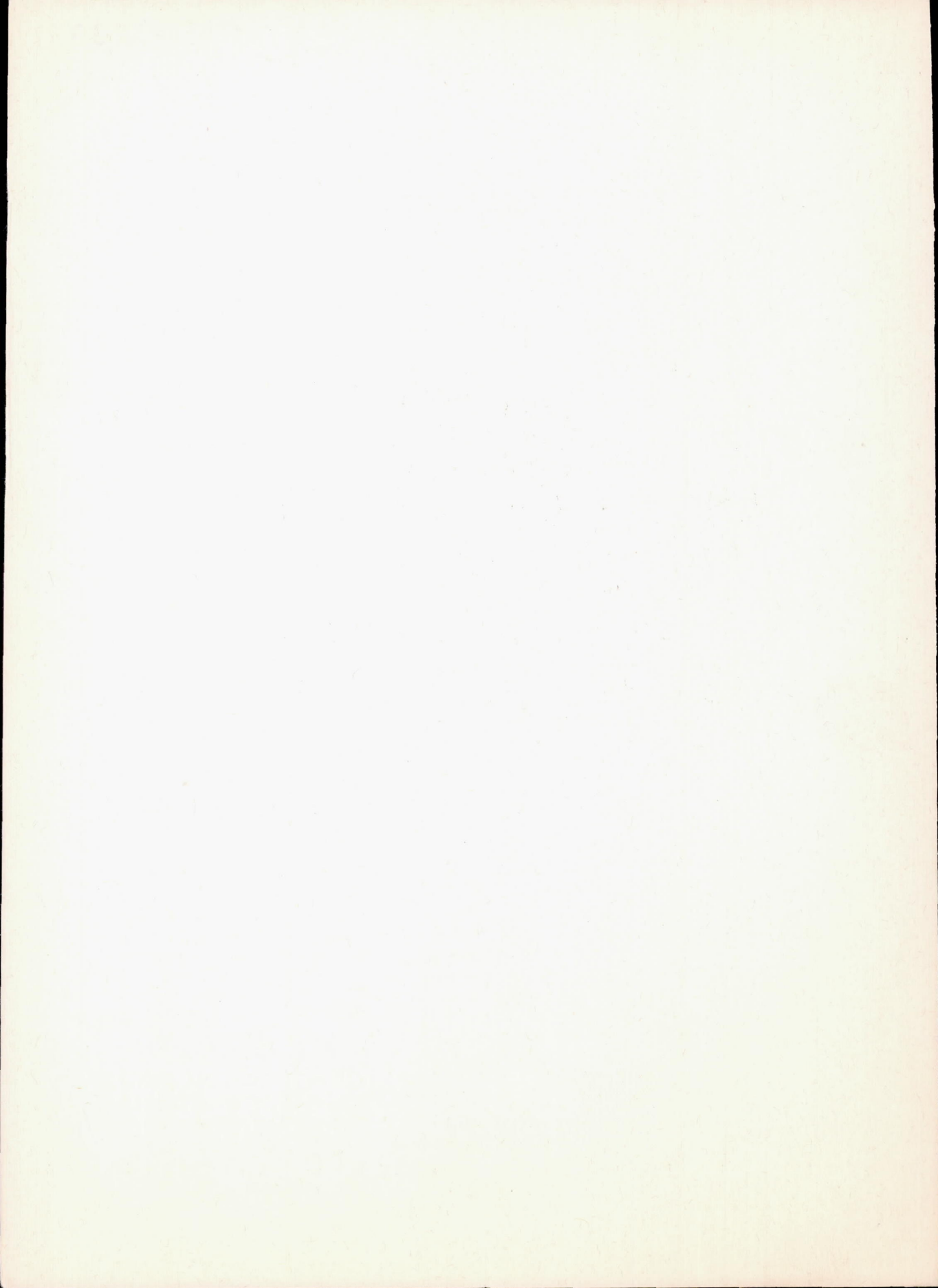
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

MEMORANDUM REPORT

for the

Army Air Forces, Materiel Command

NOTE ON THE INTERPRETATION OF WAKE-SURVEY DATA AND ITS

USE IN THE ESTIMATION OF INDUCED DRAG

DUE TO IRREGULARITIES

By S. Katzoff and Robert S. Finn

SUMMARY

An analysis of the irregular vertical displacement of the wake from the trailing edge of a tailless airplane is presented, which shows the existence of an effective aerodynamic wing twist that causes an induced drag coefficient of 0.0004. The analysis explains most of the discrepancy between the drag determined from the momentum surveys and the drag measured on the wind-tunnel balance.

INTRODUCTION

In a study of a tailless airplane in the NACA full-scale tunnel, the drag coefficient determined from a complete wake survey behind the airplane was found to be 0.0007 less than that determined from the balance readings, although the lift coefficient, -0.02 , was so small that the corresponding induced drag coefficient was quite negligible and the wing presumably had no aerodynamic twist. Investigation of the wake surveys showed, however, that regions of considerable downwash and upwash existed at certain spanwise positions, as indicated by the fact that the wake at these positions was considerably displaced above or below the level of the trailing edge. It was assumed, from this irregularity, that some aerodynamic twist existed in the wing, either in the structure itself or as a result of unsymmetrical surface roughnesses leading to differences in the boundary layer on the two surfaces. Analysis of the surveys showed that the induced drag arising from this irregularity was of the right order to account for the observed discrepancy. The analysis is briefly summarized herein.

WAKE ANALYSIS

Theory.- If, for simplification, the air density and the free-stream velocity are assumed to be unity, the induced drag (kinetic energy per unit length of trailing vortex sheet) is given by the following equation (reference 1), evaluated in a plane perpendicular to the wind direction:

$$D_i = -\frac{1}{2} \int_c \phi \frac{\partial \phi}{\partial n} ds$$

where

- D_i induced drag
 ϕ velocity potential on trailing vortex sheet
 n outward normal from trailing vortex sheet
 s distance along surface of vortex sheet

and the integral is taken completely around the sheet. The corresponding induced drag coefficient is

$$C_{D_i} = \frac{D_i}{\frac{1}{2}S} = -\frac{1}{S} \int_c \phi \frac{\partial \phi}{\partial n} ds$$

where S is the wing area. For purposes of calculation, it may be assumed that the wake is essentially plane, so that s may be taken as the spanwise distance and n may be taken as the vertical distance.

The value of $\partial \phi / \partial n$, essentially the vertical velocity of the wake, is assumed to be given by the vertical displacement of the wake divided by the longitudinal distance of the survey point from the trailing edge. In order to evaluate ϕ , the stream function ψ at every point is first obtained by integrating the normal component of the velocity $\partial \phi / \partial n$:

$$\psi(s) = \int_0^s \frac{\partial \phi}{\partial n} ds$$

The function $\psi(s)$ is now transformed to $\psi(\theta)$ by the change of variable

$$s = \frac{b}{2} (1 - \cos \theta)$$

where b is the span and s makes the complete circuit of the wake as θ goes from 0 to 2π . If ψ is now obtained in the form of a Fourier series in θ , the velocity potential ϕ will be given immediately as the conjugate Fourier series (reference 2). The induced drag coefficient is then obtainable directly by integrating the product $\phi \frac{\partial \phi}{\partial n}$ around the wake.

The symmetry of the functions $\frac{\partial \phi}{\partial n}$, ψ , and ϕ may be noted: Both $\partial \phi / \partial n$ and ϕ have the same numerical values, with opposite sign, at corresponding points above and below the wake; and ψ is the same, with regard to both numerical value and sign, at corresponding points.

Procedure.— A spanwise plot of the vertical location of the wake center is shown in figure 1, together with the location of the trailing edge ahead of it. From the displacement and the longitudinal distance from the trailing edge to the plane of the surveys (about 0.25 chord), $\partial \phi / \partial n$ was obtained (fig. 2). Numerical integration of $\partial \phi / \partial n$ across the span, starting at the left edge, gave ψ , which is shown plotted against θ in figure 3. Since ψ is symmetrical with respect to $\theta = \pi$ (corresponding to the right edge of the wake), only half of the curve is shown. The Fourier analysis of ψ and the synthesis of ϕ from the conjugate Fourier series were done with the aid of Runge's computation forms (reference 3). The velocity potential ϕ plotted against s is shown in figure 4, and the product

$\phi \frac{\partial \phi}{\partial n}$ plotted against s is shown in figure 5. The area of the product curve corresponds to a drag coefficient of 0.0004.

The only difficulty in this procedure resulted from the fact that an exact value was not available for the vertical location of the survey mechanism relative to the vertical location of the wing; that is, the relative

vertical locations of the two curves of figure 1 were not accurately known. A simple criterion exists, however, for correcting the inaccuracy; namely, the integral of ϕ across the span must correspond to the observed lift coefficient.

$$C_L = \frac{4}{\pi} \int_0^b \phi \, ds$$

Furthermore the criterion and correction are easily applied, because the effects of a uniform vertical displacement on the values of $\partial\phi/\partial n$, ψ , and ϕ at every point are additive. In the present instance a displacement of only 0.2 inch from the originally assumed position was required to obtain agreement with the observed lift coefficient.

DISCUSSION

The value derived for the induced drag coefficient is somewhat less than the value of 0.0007 sought; the two values differ, however, by little more than the experimental error in the tunnel measurements. The analysis accounts, then, for most of the discrepancy between the drag coefficients obtained from the balance measurements and from the momentum surveys.

Certain essential inaccuracies are present in the analysis. An obvious lack of rigor exists in the initial assumptions that the downwash angle is uniform between the trailing edge and the station of measurement and that it is equal, in this region, to its theoretical ultimate value. Some further inaccuracy results from the assumption that the wake origin is at the trailing edge because, if the boundary layers on the upper and lower wing surfaces are not equally thick, the wake origin will be displaced toward the thicker one. Accordingly the wake displacement, as obtained from figure 1, may be in error at any point by 0.2 or 0.3 inch. The corresponding increase in the calculated induced drag would be relatively small. In any case, the small sharp irregularities of the $\partial\phi/\partial n$ curve, some of which may have been due to such error in locating the wake origin, were largely washed out in the 24-point method

used for the Fourier analysis, so that most of the small induced drag associated with such irregularities was not actually included in the answer.

Figure 6 shows the spanwise angle-of-attack distribution corresponding to the values of ϕ and $\partial\phi/\partial n$:

$$c_l = \frac{4}{c} \phi = 0.1 \left(\alpha + \frac{57.3}{2} \frac{\partial\phi}{\partial n} \right)$$

or

$$\alpha = \frac{40\phi}{c} - \frac{57.3}{2} \frac{\partial\phi}{\partial n}$$

where

α effective geometric angle of attack, degrees

c local chord

c_l section lift coefficient

and the two-dimensional slope of the lift curve for the section is taken as 0.1 per degree.

The high angle of attack at the center of the wing probably results from the presence of the unsymmetrical bulge that constituted the fuselage. The pronounced twist of the right wing seems too large to be due to normal inaccuracies in the construction, although it might have resulted from excessive warping of the wooden forms after construction. It is also possible that, where the boundary layer on the upper surface has, owing to local surface conditions, a different thickness from the boundary layer on the lower surface, there will be a change in the effective incidence. Some evidence of such dissymmetry was given by the surveys themselves, which showed large and irregular spanwise variations of section drag coefficient; it is reasonable to suppose that the roughness causing such variations would not, at every section, be equally distributed on the upper and lower surfaces.

Inspection of the wake surveys taken behind two other wings of more rigid construction failed to show pronounced irregularities in the vertical location of the wake; it is possible that the effect discussed herein

is not a common one. It may be well, however, when wake surveys are made, to plot the vertical location, as in figure 1, before assuming that the momentum measurements should check the force tests.

CONCLUSIONS

Analysis of the irregular vertical displacement of the wake from the trailing edge of a tailless airplane showed that the induced drag due to wing irregularities contributed 0.0004 to the drag coefficient. The analysis explained most of the discrepancy between the drag determined from the momentum surveys and the drag measured on the wind-tunnel balance.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 1, 1944.

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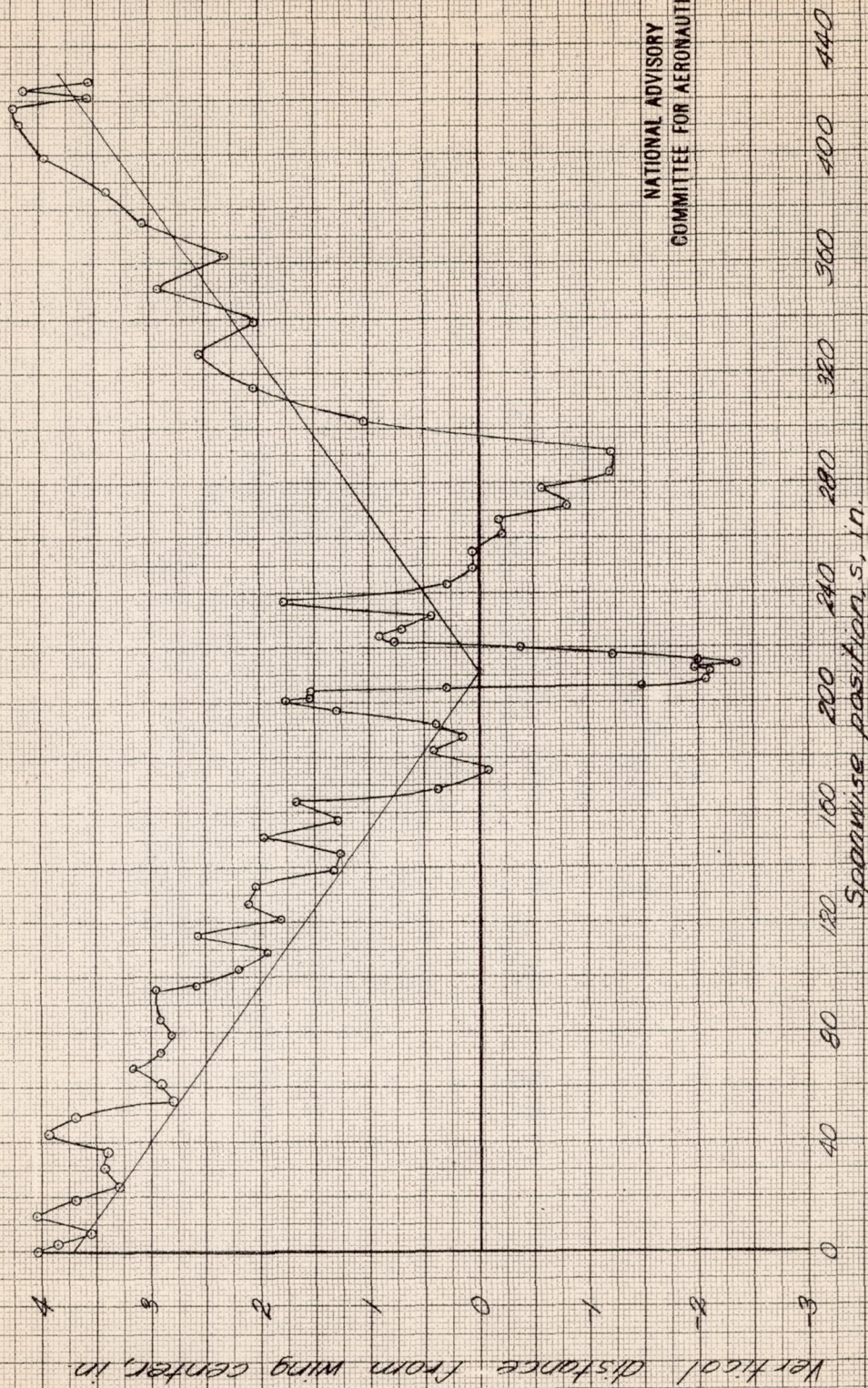
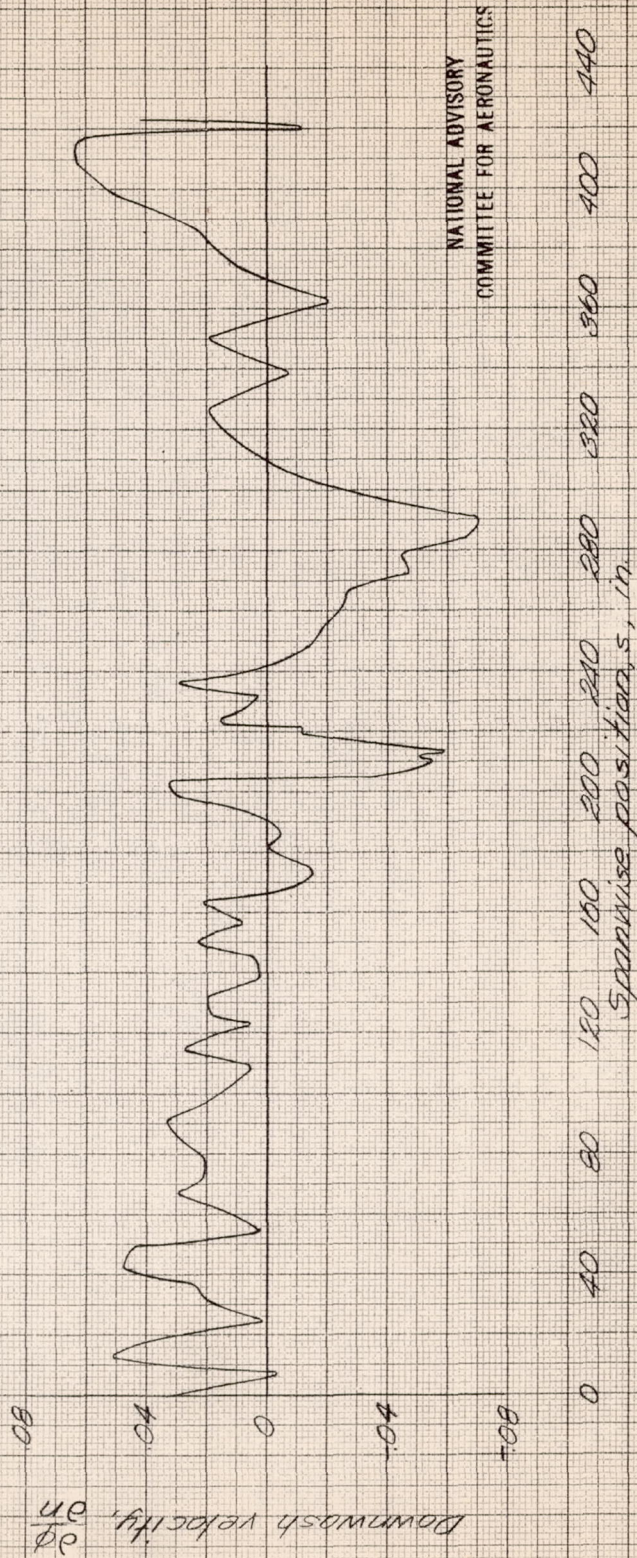


Figure 1.- Vertical locations of trailing edge and of wake center of 0.85 chord behind trailing edge, showing relative displacement.



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Figure 2.- Spanwise distribution of downwash velocity. Free-stream velocity assumed unity

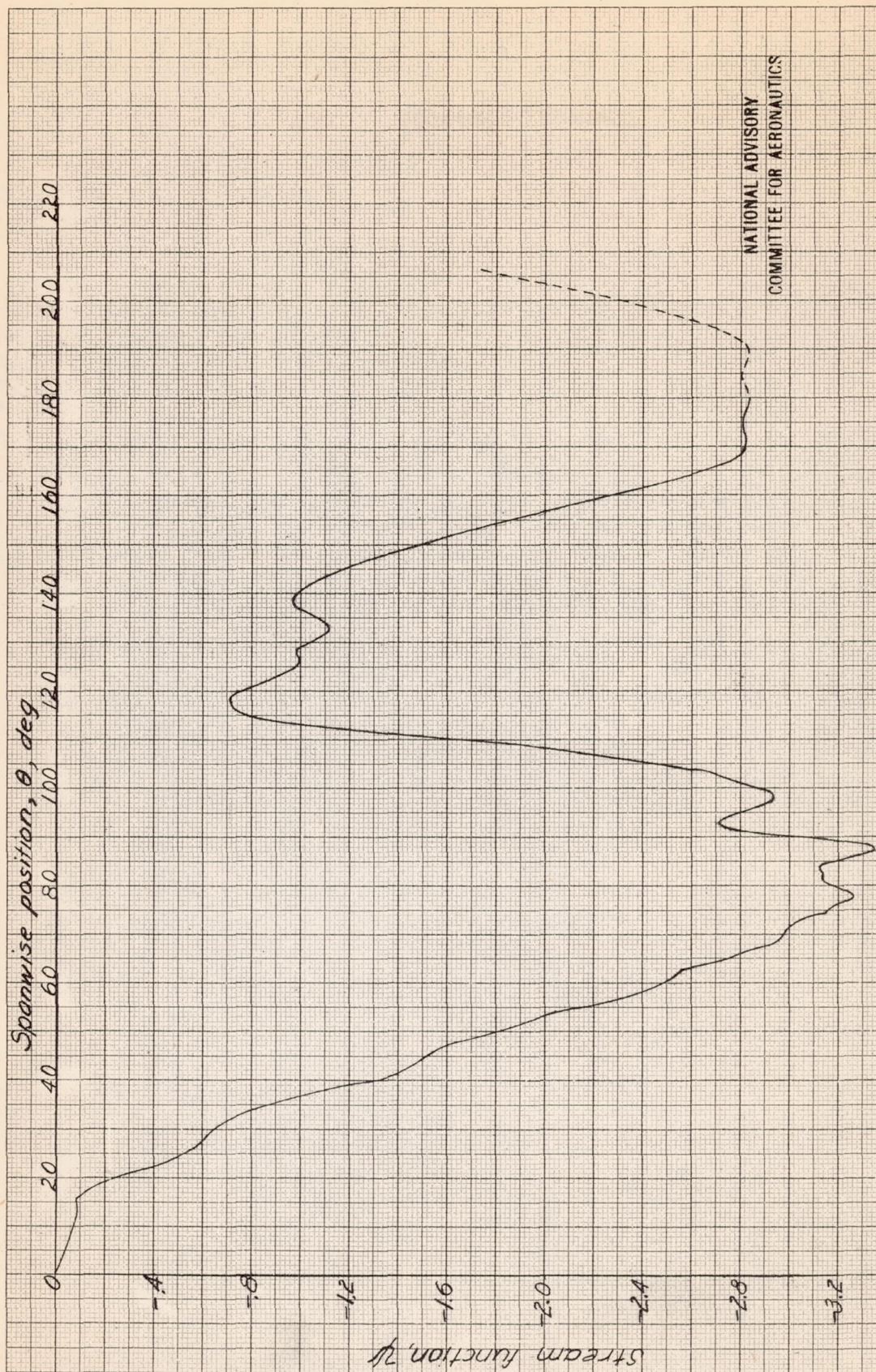
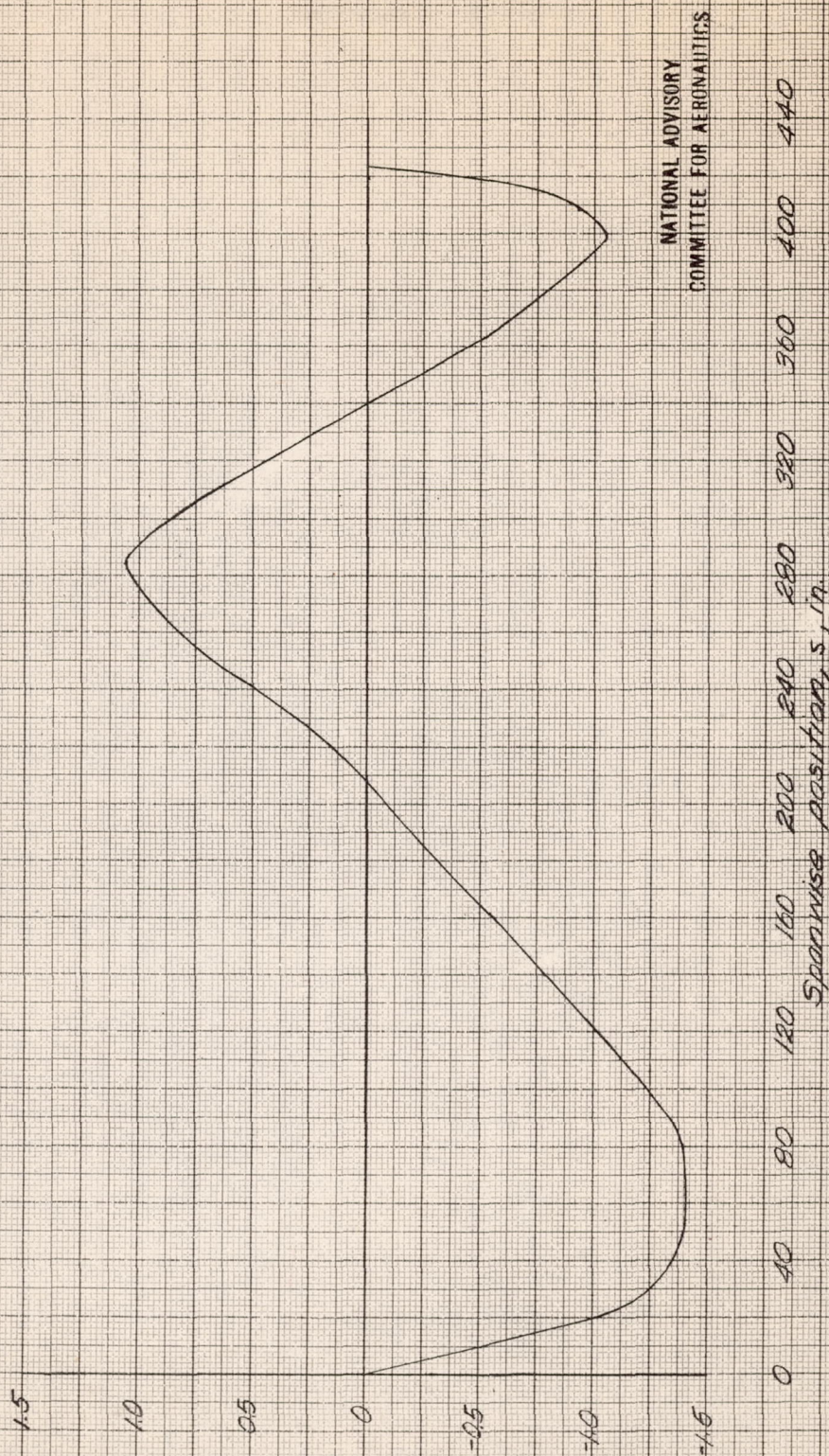


Figure 3. - Stream function ψ against θ . The distribution is symmetrical about $\theta = 180^\circ$.

Velocity potential, ϕ , on upper surface of wake



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Figure 4 - Velocity potential ϕ against spanwise position for the upper surface of the wake. For the lower surface, ϕ is the same but of opposite sign.

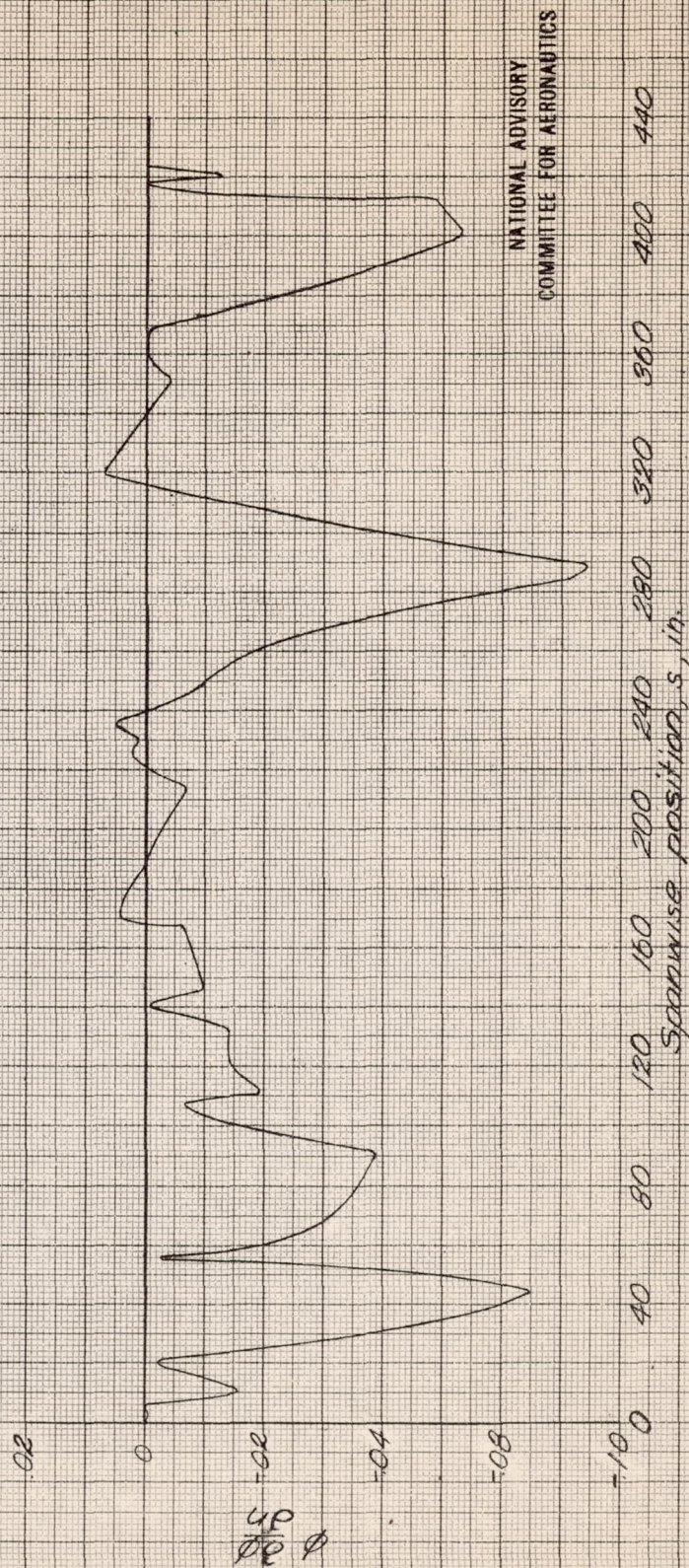


Figure 5.- Spanwise distribution of $\phi \frac{\partial \phi}{\partial y}$.

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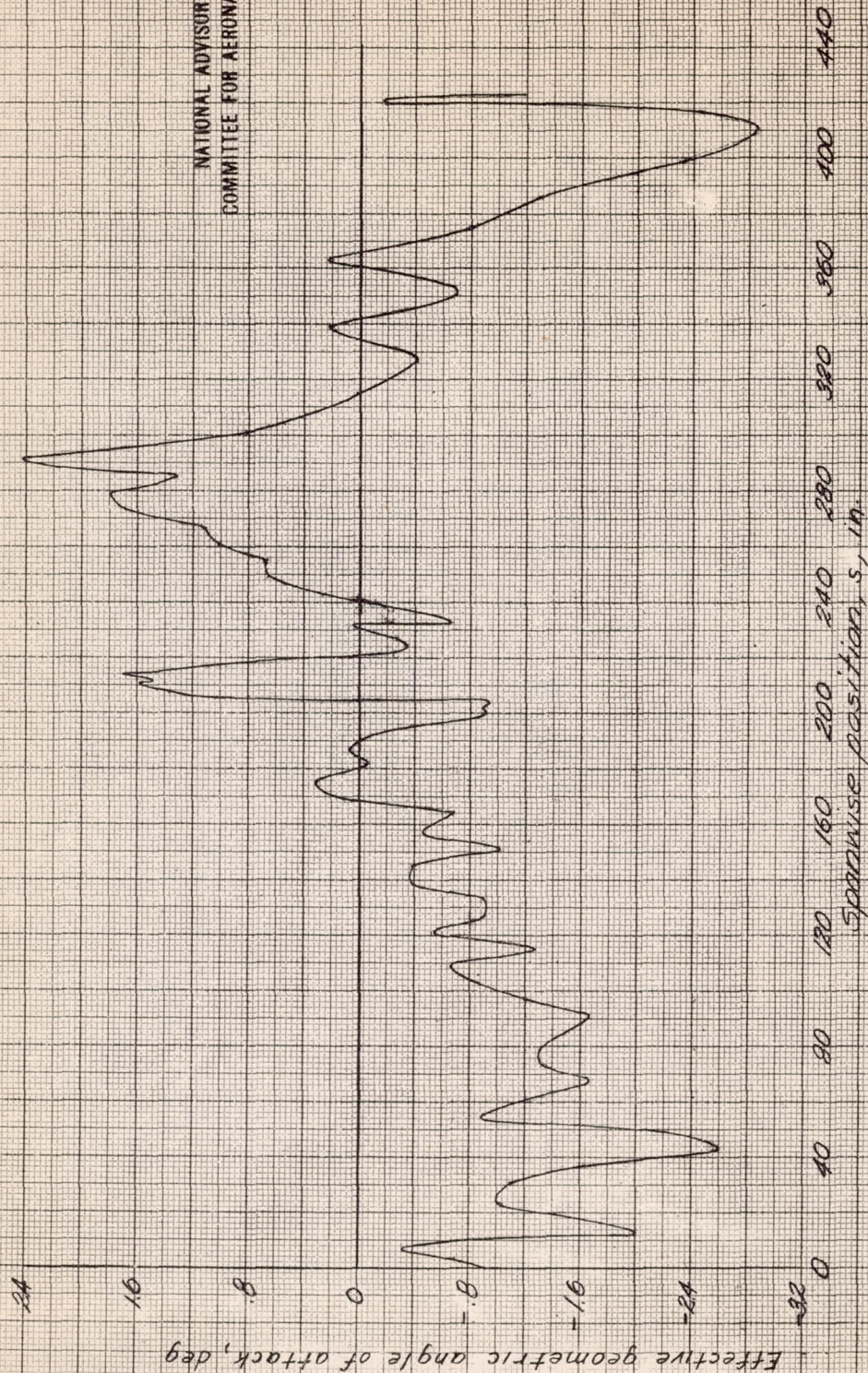


Figure 6. - Spanwise distribution of effective geometric angle of attack.